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Short Communication

A coupled method for structural damage identification

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Abstract

This paper presents a coupled method for structural damage identification. Firstly, the damage localization criterion (DLC) is defined to determine the damaged degree-of-freedom (dof). Then the damaged elements can be ascertained according to the relation between the element number and the dof number. The natural frequency sensitivity method is employed to obtain damage extents with the damaged elements determined. The presented method is demonstrated on a simply supported beam. Results show that the method is simple and effective for structural damage detection.

1. Introduction

Detection, location and quantification of damage in a structure via techniques that examine changes in measured structural vibration response have attracted much attention in recent years. The method widely used to detect damage in structures is using modal frequency changes for the lower natural frequency can be easily and precisely measured in practice. Biswas et al. demonstrated that a decrease in natural frequencies could be used to detect damage in a highway bridge [1]. Vandiver used the same principle to determine the occurrence of damage in offshore structures [2]. Messina et al. used the natural frequency sensitivity analysis to determine damage locations and extents [3,4]. These methods seem to fail to locate and quantify damages sometimes since modal frequencies are a global property of the structure, which are especially obvious for the symmetrical structure.

As mode shapes can provide much information than natural frequency, many researchers have devoted their efforts to detect damages with mode shape information or both mode shape information and natural frequency information. Pandey et al. used the changes in the mode shape curvature to detect and locate damage [5]. Mannan and Richardson located structural cracks by using the difference in the stiffness matrices of structures before and after damage [6]. Park et al. utilized a stiffness error matrix method to determine the damage locations in a structure [7]. Gysin point out that the error matrix method is effective only when all modes of the structure are included or at least those modes that are influenced most by the damage [8]. Pandey and Biswas developed a method to locate damages using changes in the flexibility matrix of the structure [9]. This approach is feasible since the structural flexibility matrix can be obtained accurately by using only a few

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of the lower frequency modes. Later they demonstrated the effectiveness of the flexibility change method using experimental data [10]. The disadvantage of their method is that results of damage localization depend on the conditions of the structural boundary. However, in actual engineering the structural boundary condition is difficult to determine ideally. Shi et al. used the change of modal strain energy in each structural element as a damage indicator before and after the occurrence of damage [11]. The elemental modal strain energy is defined as the product of the elemental stiffness matrix and the second power of the mode shape component. They improved this method to quantify damages by deriving the sensitivity of the modal strain energy with respect to a damage [12]. In general, these methods based on the mode shape information can locate the probable damages preliminary, but the absolutely accurate damage localization is impossible and the damage quantification result often has large error because the measured mode shapes include large measurement noise.

In this paper, a coupled method to identify damage is presented that combines the advantages of the above two class methods. Firstly, as an extension of the flexibility matrix change method, a damage localization technique is presented using the change of flexibility matrix and the stiffness matrix of the intact structure. The damage localization criterion (DLC) is defined to determine the damaged degree-of-freedom (dof) and then the probable damaged element can be ascertained according to the relation of the dof number and the element number. After the suspected damaged elements are determined, the natural frequency sensitivity method is employed to obtain damage extents. The presented approach is verified via an example of a simply supported beam. Results show that this coupled method is effective to detect structural damage.

2. Damage localization

Assume that the stiffness matrix and flexibility matrix for an n dofs system are K and G, then we have

$$KG = I. \tag{1}$$

Eq. (1) can be expressed as

$$\begin{bmatrix} k_1 & k_2 & \cdots & k_n \end{bmatrix} \begin{bmatrix} g_1 & g_2 & \cdots & g_n \end{bmatrix} = I, \tag{2}$$

where k_i and $g_i (i = 1 \sim n)$ are the stiffness vector and flexibility vector of the ith dof in the stiffness matrix and flexibility matrix. From Eq. (2) we obtain

$$k_i^{\mathrm{T}} g_i = 1. \tag{3}$$

For the intact and damaged structure, Eq. (3) is

$$k_{ui}^{\mathrm{T}} g_{ui} = k_{di}^{\mathrm{T}} g_{di} = 1, \tag{4}$$

where k_{ui} and g_{ui} are the stiffness vector and flexibility vector of the ith dof for the undamaged structure, while k_{di} and g_{di} are the stiffness vector and flexibility vector of the ith dof for the damaged structure. The stiffness vector is unchanged for the undamaged dof, that is

$$k_{ui} = k_{di}. \tag{5}$$

Substituting Eq. (5) into Eq. (4) yields

$$k_{ui}^{\mathrm{T}} \Delta g_i = 0, \tag{6}$$

where

$$\Delta g_i = g_{di} - g_{ui}, \tag{7}$$

Δg_i is defined as the flexibility-increased vector of the ith dof. The stiffness vector is changed for the damaged dof, i.e.,

$$k_{ui} \neq k_{di}. \tag{8}$$

Substituting Eq. (8) into Eq. (4) results in

$$k_{ui}^{\mathrm{T}} \Delta g_i \neq 0. \tag{9}$$

Eqs. (6) and (9) indicate that the damaged dof can be determined by that $k_{ui}^{\mathrm{T}}\Delta g_i$ is zero or not. We define $k_{ui}^{\mathrm{T}}\Delta g_i$ as the DLC of the ith dof. Those non-zero DLCs are associated with the damaged dofs. Now let us derive the equations to compute the flexibility-increased vector Δg_i.

With mode shapes normalized to unit mass, the flexibility matrix can be obtained approximately by using only a few of the lower frequency modes, i.e.,

$$G = \sum_{j=1}^{m} \frac{1}{\lambda_j} \phi_j \phi_j^{\mathrm{T}}, \tag{10}$$

where λ_j and ϕ_j are the jth eigenvalue and eigenvector, respectively, m is the number of measured modes. For the intact and damaged structure Eq. (10) changes to

$$G_u = \sum_{j=1}^{m} \frac{1}{\lambda_{uj}} \phi_{uj} \phi_{uj}^{\mathrm{T}}, \quad G_d = \sum_{j=1}^{m} \frac{1}{\lambda_{dj}} \phi_{dj} \phi_{dj}^{\mathrm{T}}, \tag{11,12}$$

respectively, where G_u and G_d are the flexibility matrices of the intact and damaged structure, λ_{uj}, ϕ_{uj} and λ_{dj}, ϕ_{dj} are the jth eigenvalue and eigenvector of the structure before and after damage. The flexibility matrix change can be obtained as

$$\Delta G = \sum_{j=1}^{m} \frac{1}{\lambda_{dj}} \phi_{dj} \phi_{dj}^{\mathrm{T}} - \sum_{j=1}^{m} \frac{1}{\lambda_{uj}} \phi_{uj} \phi_{uj}^{\mathrm{T}}. \tag{13}$$

As a result, Δg_i has been obtained, which is the ith column in ΔG.

For beam structures, only the transnational dofs are used in the calculation of the DLC since it is difficult to measure the rotational dofs. The DLCs with larger absolute values are associated with the damaged dofs, so we can find damaged elements according to the relation between the element number and the dof number.

3. Damage quantification

Since those damage quantification methods using mode shape information are all inevitably affected by the large measurement noise of mode shapes, a better choice to obtain the damage extent is only using the natural frequency information because the lower natural frequencies can be measured very precisely. In this paper, the natural frequency sensitivity method [3,4] is employed to identify damages after damages are located using the DLC. Without loss of generality, assuming that the number of the suspected damaged elements is r and the corresponding damage parameters are $\alpha_1, \alpha_2, \ldots, \alpha_r$, the damage quantification formulation from measured modal frequencies with the first-order approximation is as follows:

$$\begin{Bmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \\ \vdots \\ \Delta\lambda_m \end{Bmatrix} = S_f \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_r \end{Bmatrix}, \tag{14}$$

where S_f denotes the first-order sensitivity matrix of natural frequencies and $\Delta\lambda_j (j = 1 \sim m)$ is the change of the natural frequency before and after damage. From Eq. (14), the damage extents can be obtained as

$$\begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_r \end{Bmatrix} = S_f^{+} \begin{Bmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \\ \vdots \\ \Delta\lambda_m \end{Bmatrix}. \tag{15}$$

4. Numerical example

A simply supported beam structure (shown in Fig. 1) is taken as an example to verify the proposed method. The basic parameters of the structure are as follows: $E = 200\,\text{GPa}$, $\rho = 7.8 \times 10^3\,\text{kg/m}^3$, $I = 1.0416 \times 10^{-6}\,\text{m}^4$, and $A = 0.0025\,\text{m}^2$. The beam is modeled using 12 elements giving 24 dofs and the length of each element is $L = 0.1\,\text{m}$. dofs are numbered from left to right. Natural frequencies and mode shapes are contaminated with 0.1% and 5% random noises, respectively.

4.1. Single damage

Assume that a single damage occurs in the 7th element with a stiffness loss of 15%. When only the transnational dofs are measured and the first four modes are included to conduct the computation, the DLC is shown in Fig. 2. The transnational dofs numbered 6 and 7 are damaged from Fig. 2, which are exactly associated with the 7th element. Using Eq. (15) the damage extent without noise and with noise can be obtained as $\alpha_7 = 0.1686$ (12.4%) and $\alpha_7 = 0.1693$ (12.9%), respectively. The value in bracket denotes the comparative error between the calculated value and the assumed value.

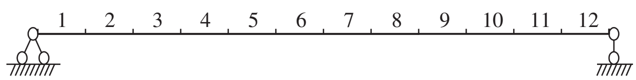

Fig. 1. A simply supported beam.

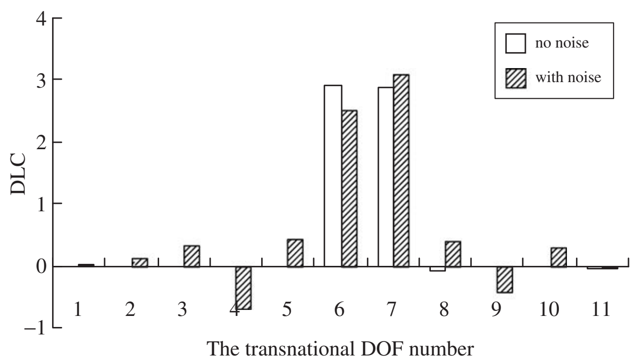

Fig. 2. DLC with element 7 damaged using incomplete modes.

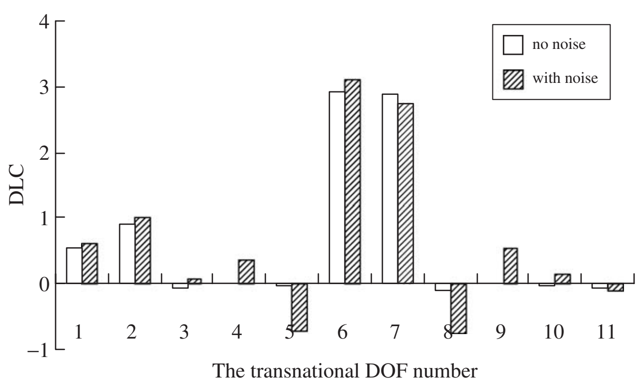

Fig. 3. DLC with elements 2 and 7 damaged using incomplete modes.

4.2. Multiple damages

Assume that two damages occur in the 2nd and 7th elements with two stiffness losses of 10% and 15%, respectively. The DLC using incomplete modes is shown in Fig. 3. The 2nd and 7th elements can be identified to be damaged elements. Again, damage extents without noise can be obtained as $\alpha_2 = 0.1115$ (11.5%) and $\alpha_7 = 0.1604$ (6.9%), respectively. When noise are considered, damage extents are $\alpha_2 = 0.1126$ (12.6%) and $\alpha_7 = 0.1642$ (9.5%), respectively. If the damage is not small, a second-order approximation on the damage [4] can be performed or an iteration scheme [12] can be used to estimate damage extents more precisely.

5. Conclusion

A coupled method for structural damage identification is presented. The approximate change of the flexibility matrix is calculated previously using a few lower modes to obtain the DLC. The DLC is then used to localize damage. With the suspected damaged element determined, the natural frequency sensitivity method is used to obtain the damage extent. The proposed method is demonstrated on a simply supported beam and results show that the presented method is simple and effective to detect structural damage.

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